INTEGRATION OF THE ERROR EQUATIONS OF AN INERTIAL GUIDANCE SYSTEM FOR KEPLERIAN MOTIONS OF AN OBJECT

(INTEGRIROVANIE URAVNENII OSHIBOK SYSTEMY INERTSIAL'NOI NAVIGATSII DLIA KEPLEROVYKH DVIZHENII OB"EKTA)

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An internal guidance system determines the position and orientation of a moving object from the readings of mass accelerometers, gyroscopic pickups of absolute angular velocity, and from specified initial conditions [1 and 2].

If the components of an inertial system have instrument errors and the initial conditions are not given exactly, the coordinates and orientation of the object will be determined inaccurately. The dependence of that inaccuracy on the instrument errors and on the inexactness of the initial conditions is described by the error equations [2] which comprise two groups of differential equations and some algebraic relations.

Below, the error equations will be integrated for an object in Keplerian motion.

1. We will introduce a right-handed system of rectangular coordinates $O_1\xi\eta\zeta$ with origin at the center of the Earth and axes invariant relative to directions f om the center of the Earth to the fixed stars.

In this system of coordinates, the error equations of an inertial guidance system have the form [2]

$$\frac{d^2\delta\mathbf{r}}{dt^2}+\frac{\mu\delta\mathbf{r}}{r^3}-\frac{\mu\mathbf{r}}{r^3}\frac{3\mathbf{r}\cdot\delta\mathbf{r}}{r^2}=\Delta\mathbf{n}-2\Delta\mathbf{m}\times\frac{d\mathbf{r}}{dt}+\mathbf{r}\times\frac{d\Delta\mathbf{m}}{dt} \qquad (1.1)$$

$$\frac{d\boldsymbol{\theta}_1}{dt}=\Delta\mathbf{m},\qquad \delta\mathbf{r}_1=\boldsymbol{\theta}_1\times\mathbf{r},\qquad \delta\mathbf{r}_2=\delta\mathbf{r}+\delta\mathbf{r}_1$$

where $\mathbf{r}$ is the radius vector from the center of the Earth O_1 to the point O of the object in which are located the sensitive masses of the accelerometers of the guidance system, $\delta\mathbf{r}$ is the change of this radius vector, μ is the product of the gravitational constant and the mass of the Earth, $\boldsymbol{\theta}_1$ is the error in orientation of the gyroscope platform of the inertial system, $\Delta\mathbf{n}$ are the instrument error of the accelerometers and $\Delta\mathbf{m}$ those of the absolute angular velocity meters, and $\delta\mathbf{r}_2$ is the total error in the coordinates of the object as determined by the inertial system.

If it is assumed that point O is the center of mass of the object, its radius vector $\mathbf{r}$ appearing in Equation (1.1) will satisfy Equation

$$\frac{d^2\mathbf{r}}{dt^2} + \frac{\mu\mathbf{r}}{r^3} = 0 \qquad (1.2)$$

The major difficulty in the integration of system (1.1) is the first equation. When the object is in Keplerian motion, the corresponding homogeneous equation can be transformed into the form

$$\delta\left(\frac{d^2\mathbf{r}}{dt^2} + \frac{\mu\mathbf{r}}{r^3}\right) = 0 \qquad (1.3)$$

i.e. it turns out to be the variation of Equation (1.2) for Keplerian motion. The general integral of Equation (1.2) containing six arbitrary constants is known. On the basis of a well-known theorem of Poincaré [3], particular solutions of the homogeneous equation (1.1) are obtained by differentiating the general integral of Equation (1.2) with respect to the arbitrary constants, thus enabling one to integrate the first equation in (1.1). In this way Lur'e [4] has integrated the vector equation for the free fall of a particle in the cabin of a satellite; this equation differes from the first equation in (1.1) only in the right-hand side.

We will introduce a trihedron $O_1\xi'\eta'\zeta'$ with the $O_1\xi'\eta'$ plane coinciding with the plane of the object orbit. The direction of the $O_1\zeta'$-axis in the normal to the orbit is such that, when looking from the end of this axis, the object moves counterclockwise. The Keplerian motion (elliptical) of the object in the plane of the orbit is determined [5] by Formulas

$$M = \nu(t - t_0) + M_0, \quad \nu = \mu^{1/2}a^{-3/2}, \quad E - e\sin E = M$$

$$r = a(1 - e\cos M), \quad \tan {}^1/_2 v = \sqrt{(1+e)/(1-e)}\,\tan {}^1/_2 E, \quad \sigma = v + \omega \qquad (1.4)$$

$$\sin E = \frac{2}{e}\sum_{k=1}^{\infty}\frac{J_k(ke)}{k}\sin kM$$

where a is the semimajor axis, e is the eccentricity of the orbit, M, E and v are, respectively, the mean, eccentric, and true anomalies, ν is the mean angular velocity of the motion, σ is the angle between the ξ'-axis and the radius-vector $\mathbf{r}$, ω is the angle between the ξ'-axis and the direction of perigee, t_0 is the time of passage through perigee, and J_k are Bessel functions.

Formulas (1.4) depend on four arbitrary constants: t_0, e, a and ω. The two remaining constants must be included in the determination of the orientation of the orbital plane relative to the coordinate system $O_1\xi\eta\zeta$.

The relative position of the trihedrons $O_1\xi\eta\zeta$ and $O_1\xi'\eta'\zeta'$ can be stipulated by the table of direction cosines:

	ξ'	η'	ζ'
ξ	$\cos\beta$	0	$\sin\beta$
η	$\sin\alpha\sin\beta$	$\cos\alpha$	$-\sin\alpha\cos\beta$
ζ	$-\cos\alpha\sin\beta$	$\sin\alpha$	$\cos\alpha\cos\beta$

(1.5)

Now

$$\mathbf{r} = \xi\boldsymbol{\xi} + \eta\boldsymbol{\eta} + \zeta\boldsymbol{\zeta}$$

$$\xi = \mathrm{r}\cos\sigma\cos\beta, \quad \eta = \mathrm{r}(\cos\sigma\sin\alpha\sin\beta + \sin\sigma\cos\alpha) \qquad (1.6)$$

$$\zeta = \mathrm{r}(-\cos\sigma\cos\alpha\sin\beta + \sin\sigma\sin\alpha)$$

where $\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta}$ are the base vectors along the axes.

Formulas (1.4) and (1.6) furnish the general integral of Equation (1.2) depending on the arbitrary constants $t_0, e, a, \omega, \alpha, \beta$. There is no loss of generality in assuming that

$$t_0 = 0, \qquad \alpha = \beta = 0, \qquad \omega = \sigma(0) = 0 \qquad (1.7)$$

in order to simplify the following notation.

We will form the following linear combinations [4] of the derivatives of the radius-vector $\mathbf{r}$ determined by Equations (1.4), (1.6)

$$\mathbf{q}_1 = \frac{\partial \mathbf{r}}{\partial a}, \quad \mathbf{q}_2 = \frac{1}{a}\frac{\partial \mathbf{r}}{\partial e}, \quad \mathbf{q}_3 = -\frac{1}{ae(1-e^2)}\frac{\partial \mathbf{r}}{\partial a} - \frac{\sqrt{1-e^2}}{aev}\frac{\partial \mathbf{r}}{\partial t_0}$$
$$\mathbf{q}_4 = \frac{1}{a}\frac{\partial \mathbf{r}}{\partial \omega}, \qquad \mathbf{q}_5 = -\frac{1}{a}\frac{\partial \mathbf{r}}{\partial \beta}, \qquad \mathbf{q}_6 = \frac{1}{a}\frac{\partial \mathbf{r}}{\partial \alpha} \qquad (1.8)$$

where the constants are arbitrary.

By $\mathbf{p}_i$ we will denote the total derivative with respect to time of vector $\mathbf{q}_i$. It is obvious that the vectors $\mathbf{q}_i$ and $\mathbf{p}_i$ form a system of particular solutions of Equation (1.2).

We will now introduce the orbital trihedron xyz whose z-axis is directed along $\mathbf{r}$ and whose y-axis coincides with $O_1\zeta'$. Then, the components $\mathbf{q}_i$ and $\mathbf{p}_i$ in this reference frame will be used to form the matrices A and B whose elements are

$$A_{1i} = \mathbf{q}_i\cdot\mathbf{x}, \qquad A_{2i} = \mathbf{q}_i\cdot\mathbf{z}, \qquad A_{3i} = \mathbf{p}_i\cdot\mathbf{x}, \qquad A_{4i} = \mathbf{p}_i\cdot\mathbf{z} \qquad (i = 1, 2, 3, 4)$$
$$B_{1i} = \mathbf{q}_{i+4}\cdot\mathbf{y}, \qquad B_{2i} = \mathbf{p}_{i+4}\cdot\mathbf{y} \qquad (i = 1, 2) \qquad (1.9)$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are the unit vectors along the respective axes.

When calculating the elements of matrices A and B, it is necessary to bear in mind the relations

$$\mathbf{p}_i = \frac{d\mathbf{q}_i}{dt}, \qquad \frac{d\mathbf{x}}{dt} = -\omega_y\mathbf{z}, \qquad \frac{d\mathbf{y}}{dt} = 0, \qquad \frac{d\mathbf{z}}{dt} = \omega_y\mathbf{x}$$
$$\omega_y = v^{\bullet} = \nu\sqrt{1-e^2}\,\frac{a^2}{r^2}, \qquad r = \frac{a(1-e^2)}{1+e\cos v}, \qquad \frac{dr}{dt} = \frac{\nu e a\sin v}{\sqrt{1-e^2}} \qquad (1.10)$$

Projecting Equation (1.1) onto the orbital trihedron axes, we arrive at two systems of scalar differential equations

$$x_1^{\cdot} = -\omega_y x_2 + x_3, \qquad x_2^{\cdot} = \omega_y x_1 + x_4 \qquad (x_1 = \delta x,\ x_2 = \delta y)$$
$$x_3^{\cdot} = -\omega_y x_4 - \mu x_1 / r^3 + \Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r \qquad (1.11)$$
$$x_4^{\cdot} = \omega_y x_3 + 2\mu x_2 / r^3 + \Delta n_z + 2r\omega_y \Delta m_y$$

$$x_5^{\cdot} = x_6,\ x_6^{\cdot} = -\mu x_5 / r^3 + \Delta n_y + 2\Delta m_x r^{\cdot} + \Delta m_x^{\cdot} r - \omega_y \Delta m_z r$$
$$(x_5 = \delta z) \qquad (1.12)$$

The elements of matrices A and B constitute a system of linearly independent particular solutions of the homogeneous systems (1.11) and (1.12), since the determinants of matrices A and B are the Wronskians of these systems [4] and are nonzero

$$|A| = -\nu^2 / 2,\ |B| = \nu\sqrt{1 - e^2}$$

The general solution of the homogeneous system (1.11) and (1.12) can now be represented in the form

$$x_i = \sum_{j=1}^{4} A_{ij} C_j \quad (i = 1, 2, 3, 4), \qquad x_i = \sum_{j=1}^{2} B_{ij} C_{4+j} \quad (i = 5, 6) \quad (1.13)$$

Then the solution of the nonhomogeneous equation can be determined by the method of variation of parameters. By introducing the matrices $D = A^{-1}$, $G = B^{-1}$ and changing back from x_1, x_2, x_5 to δx, δy, δz, we arrive at the following expressions for the components δx, δy, δz, of vector $\delta \mathbf{r}$ in the orbital trihedron:

$$\delta x = \sum_{i=1}^{4} A_{1i} \left[\int_0^t [(\Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r) D_{i3} + \right.$$
$$\left. + (\Delta n_z + 2r\omega_y \Delta m_y) D_{i4}]\, dt + \sum_{j=1}^{4} D_{ij}^{\circ} x_j^{\circ}\right] \qquad (1.14)$$

$$\delta z = \sum_{i=1}^{4} A_{2i} \left[\int_0^t [(\Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r) D_{i3} + \right.$$
$$\left. + (\Delta n_z + 2r\omega_y \Delta m_y) D_{i4}]\, dt + \sum_{j=1}^{4} D_{ij}^{\circ} x_j^{\circ}\right]$$

$$\delta y = \sum_{i=1}^{2} B_{1i} \left[\int_0^t (\Delta n_y + 2\Delta m_x r^{\cdot} + \Delta m_x^{\cdot} r - \omega_y \Delta m_z r) G_{i2} dt + \sum_{j=1}^{2} G_{ij}^{\circ} x_{j+4}^{\circ}\right]$$

The elements of matrices A, B and D, G appearing in (1.14) have been calculated in [4]

On account of (1.7), they assume the form

$$A_{11} = \frac{-3\nu t\,(1+e\cos v)}{2\sqrt{1-e^2}}, \qquad A_{12} = \frac{2+e\cos v}{1+e\cos v}\sin v$$

$$A_{13} = \frac{2+e\cos v}{1+e\cos v}\cos v, \qquad A_{14} = \frac{r}{a}$$

$$A_{21} = \frac{r}{a} - \frac{3\nu te\sin v}{2\sqrt{1-e^2}}, \qquad A_{22} = -\cos v, \qquad A_{23} = \sin v, \qquad A_{24} = 0$$

$$B_{11} = \frac{r}{a}\cos v, \qquad B_{12} = \frac{r}{a}\sin v$$

$$D_{13} = \frac{2\,(1+e\cos v)}{\nu\sqrt{1-e^2}}, \qquad D_{23} = \frac{\sqrt{1-e^2}}{\nu}\,\frac{e+2\cos v+e\cos^2 v}{1+e\cos v}$$

$$D_{33} = \frac{1}{\nu}\left[-\frac{\sqrt{1-e^2}}{1+e\cos v}(2+e\cos v)\sin v + \frac{3\nu t}{1-e^2}(1+e\cos v)\,e\right]$$

$$D_{43} = \frac{1}{\nu}\left[-e\sin v\,\frac{2+e\cos v}{\sqrt{1-e^2}\,(1+e\cos v)} + \frac{3\nu t}{(1-e^2)^2}(1+e\cos v)\right]$$

$$D_{14} = \frac{2e\sin v}{\nu\sqrt{1-e^2}}, \qquad D_{24} = \frac{\sqrt{1-e^2}}{\nu}\sin v$$

$$D_{34} = \frac{1}{\nu}\left[\frac{3\nu t}{1-e^2}e^2\sin v - \frac{\sqrt{1-e^2}}{1+e\cos v}(2e-\cos v-e\cos v)\right]$$

$$D_{44} = \frac{1}{\nu}\left[\frac{3\nu t}{(1-e^2)^2}e\sin v + \frac{e\cos v+e^2\cos^2 v-2}{\sqrt{1-e^2}\,(1+e\cos v)}\right]$$

$$G_{12} = -\frac{\sqrt{1-e^2}\sin v}{\nu(1+e\cos v)}, \qquad G_{22} = \frac{\sqrt{1-e^2}\cos v}{\nu(1+e\cos v)} \tag{1.15}$$

For C_i°, C_{i+4}° we obtain Expressions

$$C_i^\circ = \sum_{j=1}^{4} D_{ij}^\circ x_j^\circ, \qquad C_{i+4}^\circ = \sum_{j=1}^{2} G_{ij}^\circ x_{i+4}^\circ \tag{1.16}$$

The quantities $D_{i3}^\circ, D_{i4}^\circ, G_{12}^\circ, G_{22}^\circ$ can be found from (1.15), if it is assumed in the latter that $t=0$ and $v=0$. The quantities $D_{i1}^\circ, D_{i2}^\circ, G_{i1}^\circ$ [4] are equal to

$$D_{11}^\circ = D_{21}^\circ = 0, \qquad D_{31}^\circ = 1, \qquad D_{41}^\circ = -(1-e^2)^{-1/2}$$

$$D_{12}^\circ = 2/(1-e^2), \quad D_{22}^\circ = (1+e)/(1-e), \quad D_{32}^\circ = D_{42}^\circ = 0$$

$$G_{11}^\circ = 1/(1+e), \qquad G_{21}^\circ = 0 \tag{1.17}$$

In (1.16) we have, in accordance with (1.11), (1.12) and (1.10),

$$x_1^\circ = \delta x^\circ, \quad x_2^\circ = \delta z^\circ, \quad x_3^\circ = \delta x^{\circ\bullet} + \nu(1-e^2)^{-1/2}\delta z^\circ$$

$$x_4^\circ = \delta z^{\circ\bullet} - \nu(1-e^2)^{-1/2}\,\delta x^\circ, \quad x_5^\circ = \delta y^\circ, \quad x_6^\circ = \delta y^{\circ\bullet} \tag{1.18}$$

where δx°, δy°, δz°, $\delta x^{\circ\cdot}$, $\delta y^{\circ\cdot}$, $\delta z^{\circ\cdot}$ are the initial values of the variables in question.

When $e = 0$, i.e. for a circular orbit

$$r = a, \quad r^\cdot = 0, \quad \nu = \omega_0, \quad v = \omega_0 t, \quad \omega_y = \omega_0, \quad \omega_y^\cdot = 0$$

and Formulas (1.14) reduce to those found in [6].

2. The solution of the second equation in (1.1) is obvious:

$$\theta_1 = \int_0^t \Delta \mathbf{m}\, dt + \theta_1^\circ \tag{2.1}$$

Projecting on the xyz-axes, it assumes [7] the form

$$\theta_{1x} = -\sin\sigma\left[\int_0^t (-\Delta m_x \sin\sigma + \Delta m_z \cos\sigma)\, dt + \theta_{1z}^\circ\right] + \\ + \cos\sigma\left[\int_0^t (\Delta m_x \cos\sigma + \Delta m_z \sin\sigma)\, dt + \theta_{1x}^\circ\right]$$

$$\theta_{1y} = \int_0^t \Delta m_y dt + \theta_{1y}^\circ \tag{2.2}$$

$$\theta_{1z} = \cos\sigma\left[\int_0^t (-\Delta m_x \sin\sigma + \Delta m_z \cos\sigma)\, dt + \theta_{1z}^\circ\right] + \\ + \sin\sigma\left[\int_0^t (\Delta m_x \cos\sigma + \Delta m_z \sin\sigma)\, dt + \theta_{1x}^\circ\right]$$

Formulas (2.2) determine the errors of orientation relative to the $\xi\eta\zeta$-trihedron. The errors of orientation relative to the orbital trihedron can be found from Equation

$$\theta_x = r^{-1}\delta y, \qquad \theta_y = -r^{-1}\delta x, \qquad \theta_z = \theta_{1z} \tag{2.3}$$

where δx and δy are given by (1.14).

3. In Sections 1 and 2 we obtained the solutions for δx, δy, δz, θ_{1x}, θ_{1y}, θ_{1z} by quadratures. For orbits with small eccentricity and constant values of Δn_x, Δn_y, Δn_z, Δm_x, Δm_y, Δm_z one can easily obtain the first terms of the power series expansions in e for the solutions of the first and second equations in (1.1).

To the first order of accuracy in e it follows from the last equation in (1.4) that

$$\sin E = \sin \nu t\,(1 + e\cos\nu t), \qquad \cos E = \cos\nu t - e\sin^2\nu t \tag{3.1}$$

and from the fifth and sixth equations in (1.10) with the aid of (1.7) we obtain

$$r = a(1 - e\cos\nu t), \qquad \omega_y = \nu(1 + 2e\cos\nu t), \qquad \sigma = v = \nu t + 2e\sin\nu t$$
$$\sin\sigma = \sin v = \sin\nu t + e\sin 2\nu t, \qquad \cos\sigma = \cos v = \cos\nu t - 2e\sin^2\nu t \tag{3.2}$$

Relations (3.1), (3.2) lead to the following expressions for the elements of the first two rows of matix A and the last two columns of matrix D :

$$\begin{aligned}
&A_{11} = -{}^3/_2\nu t(1 + e\cos\nu t), \qquad A_{12} = 2\sin\nu t + {}^3/_2 e\sin 2\nu t\\
&A_{13} = 2\cos\nu t - e(1 + 3\sin^2\nu t), \qquad A_{14} = 1 - e\cos\nu t\\
&A_{21} = 1 - e(\cos\nu t + {}^3/_2\nu t\sin\nu t), \qquad A_{22} = -\cos\nu t + 2e\sin^2\nu t\\
&A_{23} = \sin\nu t + e\sin 2\nu t, \qquad A_{24} = 0\\
&D_{13} = 2\nu^{-1}(1 + e\cos\nu t), \qquad D_{14} = 2e\nu^{-1}\sin\nu t\\
&D_{23} = \nu^{-1}(2\cos\nu t - 3e\sin^2\nu t), \; D_{24} = \nu^{-1}(\sin\nu t + e\sin 2\nu t)\\
&D_{33} = \nu^{-1}[-2\sin\nu t + e(3\nu t - {}^3/_2\sin 2\nu t)]\\
&D_{34} = \nu^{-1}[\cos\nu t + e(\cos\nu t - 3 - \sin^2\nu t)],\\
&D_{43} = \nu^{-1}[3\nu t + e(3\nu t\cos\nu t - 2\sin\nu t)]\\
&D_{44} = \nu^{-1}[-2 + 3e(\nu t\sin\nu t + \cos\nu t)]
\end{aligned} \tag{3.3}$$

Similarly

$$\begin{aligned}
&B_{11} = \cos\nu t - e(1 + \sin^2\nu t), \qquad B_{12} = \sin\nu t + {}^1/_2 e\sin 2\nu t\\
&G_{12} = -\nu^{-1}(\sin\nu t + {}^1/_2 e\sin 2\nu t), \qquad G_{22} = \nu^{-1}[\cos\nu t - e(1 + \sin^2\nu t)]
\end{aligned} \tag{3.4}$$

Let us substitute (3.3), (3.4) and (1.19) together with the initial values D_{ij}°, G_{ij}° of the elements of matrices D and G in Formulas (1.14). When Δn_x, Δn_y, Δn_z, Δm_x, Δm_y, Δm_z are constant we obtain, after integration and simplifications, the following expressions for δx, δy, δz :

$$\begin{aligned}
\delta x = {} & \delta x^\circ + \nu^{-1}\delta x^{\circ\cdot}(4\sin\nu t - 3\nu t) + 6\delta z^\circ(\sin\nu t - \nu t) + 2\nu^{-1}\delta z^{\circ\cdot}(\cos\nu t - 1) +\\
& + \nu^{-2}\Delta n_x[-{}^3/_2(\nu t)^2 + 4(1 - \cos\nu t)] + 4\nu^{-1}a\Delta m_y(\sin\nu t - \nu t) + 2\nu^{-2}\Delta n_z(\sin\nu t - \nu t) +\\
& + e[\Delta n_x\nu^{-2}(-{}^3/_2\nu^2t^2\cos\nu t - 5\nu t\sin\nu t + \cos\nu t - 1 + 6\sin^2\nu t) +\\
& + \Delta n_z\nu^{-2}(-3\nu t - 5\nu t\cos\nu t + {}^7/_2\sin\nu t + {}^5/_2\sin 2\nu t - {}^1/_2\sin\nu t\cos 2\nu t) +\\
& + 2\Delta m_y a\nu^{-1}(-6\nu t - 6\nu t\cos\nu t + {}^{15}/_2\sin\nu t + {}^5/_2\sin 2\nu t - {}^1/_2\sin\nu t\cos 2\nu t) +\\
& + \delta x^\circ(1 - \cos\nu t) - 3\nu^{-1}\delta x^{\circ\cdot}(\nu t + \nu t\cos\nu t - \sin 2\nu t) - 3\delta z^\circ(5\nu t + 2\nu t\cos\nu t -\\
& - {}^3/_2\sin 2\nu t - 4\sin\nu t) + \nu^{-1}\delta z^{\circ\cdot}(1 + \cos^2\nu t - 2\cos\nu t)
\end{aligned}$$

$$\begin{aligned}
\delta y = {} & \nu^{-2}(\Delta n_y - a\nu\Delta m_z)(1 - \cos\nu t) + \delta y^\circ\cos\nu t + \nu^{-1}\delta y^{\circ\cdot}\sin\nu t +\\
& + e[\nu^{-2}\Delta n_y(1 - \cos\nu t + \sin^2\nu t - {}^3/_2\nu t\sin\nu t) + \nu^{-1}a\Delta m_z(\cos\nu t - 1 - \sin^2\nu t +\\
& + \nu t\sin\nu t) + \nu^{-1}a\Delta m_x(\nu t\cos\nu t - \sin\nu t) + \delta y^\circ(\cos\nu t - 1 - \sin^2\nu t) +\\
& + \nu^{-1}\delta y^{\circ\cdot}(-\sin\nu t + \sin\nu t\cos\nu t)]
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\delta z = {} & 2\nu^{-1}(\delta x^\circ + a\Delta m_y)(1 - \cos\nu t) + \delta z^\circ(4 - 3\cos\nu t) + \nu^{-1}\delta z^{\circ\cdot}\sin\nu t +\\
& + 2\nu^{-2}\Delta n_x(\nu t - \sin\nu t) + \nu^{-2}\Delta n_z(1 - \cos\nu t) + e[\Delta n_x\nu^{-2}(-{}^3/_2\nu^2t^2\sin\nu t -\\
& - {}^1/_2\nu t\cos\nu t + {}^9/_2\sin\nu t - 2\sin 2\nu t) + \Delta n_z\nu^{-2}(-{}^7/_2\nu t\sin\nu t + 3\sin^2\nu t -\\
& - 2 - 2\cos\nu t - {}^1/_2\sin^2\nu t\cos\nu t) + 2a\Delta m_y\nu^{-1}(-4\nu t\sin\nu t + 4 - {}^{21}/_4\cos\nu t +\\
& + 3\sin^2\nu t + {}^5/_4\cos\nu t\cos 2\nu t) + 2\nu^{-1}\delta x^{\circ\cdot}(1 - \cos\nu t - {}^3/_2\nu t\sin\nu t) +\\
& + \delta z^\circ(-6\nu t\sin\nu t + 10 - 10\cos\nu t + 6\sin^2\nu t) + \nu^{-1}\delta z^{\circ\cdot}(\sin 2\nu t - 2\sin\nu t)]
\end{aligned}$$

Similarly, from (2.2)

$$\theta_{1x} = \theta_{1x}{}^\circ \cos \nu t - \theta_{1z}{}^\circ \sin \nu t + \nu^{-1}\Delta m_x \sin \nu t - \nu^{-1}\Delta m_z (1 - \cos \nu t) +$$
$$+ e [- 2\theta_{1x}{}^\circ \sin^2 \nu t - \theta_{1z}{}^\circ \sin 2\nu t + \nu^{-1}\Delta m_x (\nu t + {}^1/_2 \sin 2\nu t) - {}^1/_2\nu^{-1}\Delta m_z (1 - \cos 2\nu t)]$$

$$\theta_{1y} = \theta_{1y}{}^\circ + \Delta m_y t \qquad (3.6)$$

$$\theta_{1z} = \theta_{1x}{}^\circ \sin \nu t + \theta_{1z}{}^\circ \cos \nu t + \nu^{-1}\Delta m_x (1 - \cos \nu t) + \nu^{-1}\Delta m_z \sin \nu t +$$
$$+ e [\theta_{1x}{}^\circ \sin 2\nu t - 2\theta_{1z}{}^\circ \sin^2 \nu t + {}^1/_2\nu^{-1}\Delta m_x (1 - \cos 2\nu t) + \nu^{-1}\Delta m_z(\nu t + {}^1/_2 \sin 2\nu t)]$$

When $e = 0$, i.e. for a circular orbit, relations (3.5) and (3.6) reduce to those obtained for this case in [6].

From the last two equations in (1.1) we find the components of vector δr_2 of the total error of determination of coordinates in the xyz reference frame

$$\delta x_2 = \delta x + \theta_{1y} r, \qquad \delta y_2 = \delta y - \theta_{1x} r, \qquad \delta z_2 = \delta z \qquad (3.7)$$

and from (2.3) the error of orientation of the object relative to the orbital trihedron.

4. The preceding consideration was related to a self-contained inertial system with three mass accelerometers. Now we can consider the case where the inertial system also makes use of signals from an external source on the magnitude r of the distance of the object from the Earth's center. Two different methods of using this additional information are of interest [2].

In the first of these methods all three mass accelerometers are retained in the system, but the term μ/r^3 in the equations for the unperturbed operation contains r which is supplied by the additional source of information. As far as the error equations are concerned, this variant differs from that considered in Sections 1 to 3 in that the first equation in (1.1) assumes the form

$$\frac{d^2\delta \mathbf{r}}{dt^2} + \frac{\mu\delta \mathbf{r}}{r^3} = \Delta\mathbf{n} - 2\Delta\mathbf{m} \times \frac{d\mathbf{r}}{dt} + \mathbf{r} \times \frac{d\Delta\mathbf{m}}{dt} + \frac{3\mu\mathbf{r}\Delta r}{r^4} \qquad (4.1)$$

where Δr now denotes the error in the value of r transmitted to the inertial system.

In the second method, only two mass accelerometers are used in the system. The value of r transmitted to the system can be used to eliminate one of the variables from the equations for the unperturbed operation. If, in the unperturbed state, the trihedron of the inertial system, along which are situated the mass accelerometers, coincides with the orbital trihedron xyz, then the system will be without the mass accelerometer n_z along the z-axis. Thus, the three second order scalar equations corresponding to the first equation in (1.1) will reduce to two obtained by projecting onto the x- and y-axes (4.2)

$$\delta x^{\cdot\cdot} + (\mu / r^3 - \omega_y{}^2)\, \delta x = \Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y{}^{\cdot} r - \omega_y{}^{\cdot}\, \Delta r - 2\omega_y \Delta r^{\cdot}$$
$$\delta y^{\cdot\cdot} + (\mu / r^3)\, \delta y = \Delta n_y + 2\Delta m_x r^{\cdot} + \Delta m_x{}^{\cdot} r - \omega_y \Delta m_z r$$

The homogeneous equations corresponding to (4.1) and (4.2) have variable coefficients, as was the case with Equation (1.1). On the other hand, contrary to Equation (1.1), Equations (4.1) and (4.2) will not be the equations for the variations of the Keplerian motion (1.2). Thus, Poincaré's theorem can not be employed in order to search for the solutions of Equations (4.1) and (4.2). Nevertheless, the general solution of these equations can be constructed.

Let us consider Equations (4.2). The second equation coincides with system (1.12). The last formula in (1.14) and the second formula (3.5) will be its solution.

In order to construct the solution of the first equation, we note that $\delta x = r / a,\ \delta z = 0$ is one of the particular solutions of system (1.11). Comparing the first equation in (4.2) with the projection on the x-axis of the first equation in (1.1), we come to the conclusion that

$$\delta x = r / a \tag{4.3}$$

will be a particular solution of the homogeneous equation (4.2), as can be verified by direct substitution.

In order to find the second particular solution, we can now make use of the Ostrogradskii-Liouville formula, which yields

$$\delta x = \frac{r}{a} \int_0^t \frac{a^2}{r^2}\, dt \tag{4.4}$$

From the fifth equation in (1.10) we have $a^2 / r^2 = v^{\cdot} / (\nu\sqrt{1 - e^2})$. On account of this relation, (4.4) assumes the form

$$\delta x = \frac{rv}{a\nu\sqrt{1 - e^2}} \tag{4.5}$$

The solutions (4.3) and (4.5) are linearly independent. The Wronskian of these solutions is equal to unity. Thus, the general solution of the homogeneous equation corresponding to the first one in (4.2) has the form

$$\delta x = C_1 \frac{r}{a} + C_2 \frac{rv}{a\nu\sqrt{1 - e^2}} \tag{4.6}$$

The general solution of the nonhomogeneous equation can now be obtained by varying the parameters C_1 and C_2 . By virtue of the initial conditions (1.18), this solution is found in the form

$$\begin{aligned}\delta x = \frac{r}{a}\Big[-\frac{1}{a\nu\sqrt{1-e^2}}\int_0^t rv\,(\Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r - \omega^{\cdot}_y \Delta r - \\ - 2\omega_y \Delta r^{\cdot})\, dt + \frac{\delta x^{\circ}}{1-e}\Big] + \frac{rv}{a\nu\sqrt{1-e^2}}\Big[\frac{1}{a}\int_0^t r\,(\Delta n_x - 2\Delta m_y r^{\cdot} - \\ - \Delta m_y^{\cdot} r - \omega_y^{\cdot}\Delta r - 2\omega_y \Delta r^{\cdot})\, dt + (1-e)\,\delta x^{\circ\cdot}\Big]\end{aligned} \tag{4.7}$$

When the instrument errors are constant and the eccentricity of the orbit is small, we obtain from (4.7) the following approximate formulas analogous to those in (3.5):

$$\delta x = {}^1/_2 \Delta n_x t^2 + \delta x^\circ + \delta x^{\circ\cdot} t + e\,[\Delta n_x \nu^{-2}(-\nu^2 t^2 \cos \nu t + 2\nu t \sin \nu t + 3\cos \nu t - 3) + $$
$$+ 2\nu^{-1}(a\Delta m_y - \nu\Delta r)(\sin \nu t - \nu t) + \delta x^\circ (1 - \cos \nu t) + \nu^{-1}\delta x^{\circ\cdot}(2\sin \nu t - \nu t - \nu t \cos \nu t)] \tag{4.8}$$

When $e = 0$, Formula (4.8) reduces to that obtained in [6] for this case.

5. A more complicated problem is posed by the integration of Equation (4.1); its projection onto the xyz-axes leads to the system

$$\delta x^{\cdot\cdot} + (\mu / r^3 - \omega_y^2)\,\delta x + \omega_y^{\cdot}\,\delta z + 2\omega_y \delta z^{\cdot} = \Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r$$
$$\delta z^{\cdot\cdot} + (\mu / r^3 - \omega_y^2)\,\delta z - \omega_y^{\cdot}\delta x - 2\omega_y \delta x^{\cdot} = \Delta n_z + 2r\omega_y \Delta m_y + 3\mu\Delta r / r^3 \tag{5.1}$$

and to Equation

$$\delta y^{\cdot\cdot} + \mu r^{-3}\delta y = \Delta n_y + 2\Delta m_x r^{\cdot} + \Delta m_x^{\cdot} r - \omega_y \Delta m_z r \tag{5.2}$$

Equation (5.2) coincides with the second equation in (4.2), and the problem thus reduces to that of solving (5.1).

The homogeneous system of equations (5.1) has two particular solutions

$$\delta x = r / a, \quad \delta z = 0, \qquad \delta x = 0, \quad \delta z = r / a \tag{5.3}$$

This enables one to reduce it to second order by making the change of variables

$$\delta x = \frac{r}{a}\int p\,dt, \qquad \delta z = \frac{r}{a}\int q\,dt \tag{5.4}$$

The equations for p and q have the form

$$p^{\cdot} + 2\frac{r^{\cdot}}{r}p + 2\omega_y q = 0, \qquad q^{\cdot} + 2\frac{r^{\cdot}}{r}q - 2\omega_y p = 0 \tag{5.5}$$

Introducing the complex variable $u = p + iq$, we are led to a first-order equation for u

$$u^{\cdot} + 2u\left(\frac{r^{\cdot}}{r} - i\omega_y\right) = 0 \tag{5.6}$$

which can be directly integrated. The general solution of this equation is given by the function

$$u = \frac{C}{r^2}(\cos 2v + i \sin 2v) \tag{5.7}$$

where C is a complex constant.

Changing back again to the variables δx, δz and thereby making use of (4.6), we obtain the following two particular solutions of system (5.1) :

$$\delta x = \frac{r}{a}\sin 2v, \qquad \delta z = \frac{r}{a}\cos 2v$$
$$\delta x = -\frac{r}{a}\cos 2v, \qquad \delta z = \frac{r}{a}\sin 2v \tag{5.8}$$

Expressions (5.8) and (5.3) constitute a system of four particular solutions of Equations (5.1).(*) (Please find Footnote on opposite page).

Now we will construct a matrix α from the particular solutions (5.3) and (5.8) and their derivatives, the latter being related by virtue of the fifth and seventh equations in (1.10).

The elements of matrix α are

$$\alpha_{11} = \frac{r}{a}, \quad \alpha_{12} = 0, \quad \alpha_{13} = \frac{r}{a}\sin 2v, \quad \alpha_{14} = \frac{r}{a}\cos 2v \tag{5.9}$$

$$\alpha_{21} = 0, \quad \alpha_{22} = \frac{r}{a}, \quad \alpha_{23} = -\frac{r}{a}\cos 2v, \quad \alpha_{24} = \frac{r}{a}\sin 2v$$

$$\alpha_{31} = \frac{ve}{\sqrt{1-e^2}}\sin v, \quad \alpha_{33} = \frac{ve}{\sqrt{1-e^2}}\sin v \sin 2v + \frac{2av\sqrt{1-e^2}}{r}\cos 2v$$

$$\alpha_{32} = 0, \quad \alpha_{34} = \frac{ve}{\sqrt{1-e^3}}\sin v \cos 2v - \frac{2av\sqrt{1-e^2}}{r}\sin 2v$$

$$\alpha_{41} = 0, \quad \alpha_{43} = -\frac{ve}{\sqrt{1-e^2}}\sin v\cos 2v + \frac{2av\sqrt{1-e^2}}{r}\sin 2v$$

$$\alpha_{42} = \frac{ve}{\sqrt{1-e^2}}\sin v \quad \alpha_{44} = \frac{ve}{\sqrt{1-e^2}}\sin v\sin 2v + \frac{2av\sqrt{1-e^2}}{r}\cos 2v$$

The determinant of this matrix is the Wronskian of the obtained system of particular solutions.

If we reduce the homogeneous system (5.1) to the Cauchy form, the matrix of the right-hand sides will not contain diagonal elements, as was the case with Equations (1.11). Therefore, it follows from the well-known theorem of Ostrogradskii-Liouville that the Wronskian is constant. It suffices to calculate its value at $t = 0$. Since then $v = 0$, it immediately follows from (5.9) that $|\alpha| = 4v^2(1-e^2) \neq 0$.

Thus, the above particular solutions of the homogeneous system of equations (5.1) are linearly independent; the general solution of the homogeneous system

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*) The particular solutions (5.3) of Equations (5.1) have been found by first comparing systems (5.1) and (1.11) and then selecting from the particular solutions of the latter. While reviewing the manuscript of this paper, Lur'e pointed out a direct method of obtaining the general solution of the homogeneous vector equation (4.1).

From (1.2) and the homogeneous equations (4.1) it follows that

$$\mathbf{r}^{\cdot\cdot} \times \delta\mathbf{r} + \delta\mathbf{r}^{\cdot\cdot} \times \mathbf{r} = (\mathbf{r}^{\cdot} \times \delta\mathbf{r} + \delta\mathbf{r}^{\cdot} \times \mathbf{r})^{\cdot} = 0$$

$$\mathbf{r}^{\cdot} \times \delta\mathbf{r} + \delta\mathbf{r}^{\cdot} \times \mathbf{r} = \mathbf{a} \qquad (\mathbf{a} \text{ is a constant vector})$$

where the dots now indicate total time derivatives. Whence one finds

$$\delta\mathbf{r}^{\cdot}\mathbf{r}^2 - \delta\mathbf{r}\mathbf{r}\cdot\mathbf{r}^{\cdot} = \mathbf{r}\delta\mathbf{r}^{\cdot}\cdot\mathbf{r} + \mathbf{r}\times\mathbf{a} - \mathbf{r}^{\cdot}\mathbf{r}\cdot\delta\mathbf{r} \tag{1}$$

Moreover, from (1.2) and (4.1)

$$\mathbf{r}\cdot\delta\mathbf{r}^{\cdot\cdot} - \mathbf{r}^{\cdot\cdot}\cdot\delta\mathbf{r} = 0, \qquad \mathbf{r}\cdot\delta\mathbf{r}^{\cdot} - \mathbf{r}^{\cdot}\cdot\delta\mathbf{r} = c = \text{const}$$

Substitution into (1) yields (note that $\mathbf{r}\cdot\mathbf{r}^{\cdot} = rr^{\cdot}$)

$$\delta\mathbf{r}^{\cdot}r^2 - \delta\mathbf{r}rr^{\cdot} = \delta\mathbf{r}\times(\mathbf{r}\times\mathbf{r}) + c\mathbf{r} + \mathbf{r}\times\mathbf{a}$$

But $\mathbf{r}\times\mathbf{r}^{\cdot} = r^2v^{\cdot}\mathbf{y}$. Therefore, we arrive at Equation

$$\delta\mathbf{r}^{\cdot} - \delta\mathbf{r}r^{\cdot}/r + v^{\cdot}\mathbf{y}\times\delta\mathbf{r} = (c\mathbf{r} + \mathbf{r}\times\mathbf{a})/r^2$$

which is equivalent to Equations

$$(\delta z^{\cdot} + i\delta x^{\cdot}) - (r^{\cdot}/r - 2iv^{\cdot})(\delta z + i\delta x) = (c - ia_y)/r, \qquad \delta y^{\cdot} - \delta y r^{\cdot}/r = a_x/r$$

Integration yields

$$\delta z + i\delta x = (c_1 + ic_2)re^{-2iv} + (c - ia_y)/2ir^2v^{\cdot}, \qquad \delta y = c_3 r + a_x v/rv$$

The solution so obtained contains all six parameters $(c, a_y, a_x, c_1, c_2, c_3)$.

will be

$$\delta x = \sum_{i=1}^{4} C_i \alpha_{1i}, \qquad \delta z = \sum_{i=1}^{4} C_i \alpha_{2i} \tag{5.10}$$

The general solution of the nonhomogeneous system will be found by the method of variation of parameters. Assuming that the C_i are time dependent, they will be determined by Equations

$$\begin{aligned} &\sum_{i=1}^{4} C_i^{\cdot}(t)\,\alpha_{1i} = 0, \qquad \sum_{i=1}^{4} C_i^{\cdot}(t)\,\alpha_{2i} = 0 \\ &\sum_{i=1}^{4} C_i^{\cdot}(t)\,\alpha_{3i} = \Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r \\ &\sum_{i=1}^{4} C_i^{\cdot}(t)\,\alpha_{4i} = \Delta n_z + 2r\Delta m_y \omega_y + 3\mu \frac{\Delta r}{r^3} \end{aligned} \tag{5.11}$$

The elements of the matrix $\beta = \alpha^{-1}$ are found to be

$$\beta_{11} = \frac{a}{r}, \quad \beta_{12} = \frac{e \sin v}{2(1-e^2)}, \quad \beta_{13} = 0, \quad \beta_{14} = -\frac{r}{a}\frac{1}{2\nu\sqrt{1-e^2}}$$

$$\beta_{21} = -\frac{e \sin v}{2(1-e^2)}, \quad \beta_{22} = \frac{a}{r}, \quad \beta_{23} = \frac{r}{a}\frac{1}{2\nu\sqrt{1-e^2}}, \quad \beta_{24} = 0$$

$$\beta_{31} = -\frac{e \sin v \cos 2v}{2(1-e^2)}, \quad \beta_{32} = -\frac{e \sin v \sin 2v}{2(1-e^2)}, \quad \beta_{33} = \frac{r}{a}\frac{\cos 2v}{2\nu\sqrt{1-e^2}}$$

$$\beta_{34} = \frac{r}{a}\frac{\sin 2v}{2\nu\sqrt{1-e^2}}, \quad \beta_{41} = \frac{e \sin v \sin 2v}{2(1-e^2)}, \quad \beta_{42} = -\frac{e \sin v \cos 2v}{2(1-e^2)}$$

$$\beta_{43} = -\frac{r}{a}\frac{\sin 2v}{2\nu\sqrt{1-e^2}}, \qquad \beta_{44} = \frac{r}{a}\frac{\cos 2v}{2\nu\sqrt{1-e^2}} \tag{5.12}$$

With the aid of matrix β, the expressions for $C_i(t)$ can be obtained in the form

$$\begin{aligned} C_i(t) = \int_0^t [\beta_{i3}(\Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r) + \\ + \beta_{i4}(\Delta n_z + 2r\omega_y \Delta m_y + 3\mu\Delta r / r^3)]\, dt + C_i^{\circ} \end{aligned} \tag{5.13}$$

In order to obtain the solution of Equations (5.1), one must now substitute (5.13) into (5.10), after having determined the C_i° in agreement with the initial conditions. This yields

$$\begin{aligned} \delta x = \sum_{i=1}^{4} \alpha_{1i} \Big\{ \int_0^t [\beta_{i3}(\Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r) + \\ + \beta_{i4}(\Delta n_z + 2r\omega_y \Delta m_y + 3\mu\Delta r / r^3)]\, dt + C_i^{\circ} \Big\} \\ \delta z = \sum_{i=1}^{4} \alpha_{2i} \Big\{ \int_0^t [\beta_{i3}(\Delta n_x - 2\Delta m_y r^{\cdot} - \Delta m_y^{\cdot} r) + \\ + \beta_{i4}(\Delta n_z + 2r\omega_y \Delta m_y + 3\mu\Delta r / r^3)]\, dt + C_i^{\circ} \Big\} \end{aligned} \tag{5.14}$$

where

$$C_1{}^\circ = \frac{\delta x^\circ}{1-e} - \frac{\delta z^{\circ\cdot}(1-e)}{2\nu\sqrt{1-e^2}}, \qquad C_2{}^\circ = \frac{\delta z^\circ}{1-e} + \frac{\delta x^{\circ\cdot}(1-e)}{2\nu\sqrt{1-e^2}}$$
$$C_3{}^\circ = \frac{\delta x^{\circ\cdot}(1-e)}{2\nu\sqrt{1-e^2}}, \qquad C_4{}^\circ = \frac{\delta z^{\circ\cdot}(1-e)}{2\nu\sqrt{1-e^2}} \qquad (5.15)$$

When the orbit eccentricity is small, (3.2) can be used to give the following first order expressions

$$\alpha_{11} = 1 - e\cos\nu t, \quad \alpha_{12} = 0, \quad \alpha_{13} = \sin 2\nu t + e\sin\nu t\,(3\cos 2\nu t - 1)$$
$$\alpha_{14} = \cos 2\nu t - e\,(3\sin\nu t\,\sin 2\nu t + \cos\nu t), \quad \alpha_{21} = 0$$
$$\alpha_{22} = 1 - e\cos\nu t, \quad \alpha_{23} = -\cos 2\nu t + e\,(3\sin\nu t\sin 2\nu t + \cos\nu t)$$
$$\alpha_{24} = \sin 2\nu t + e\sin\nu t\,(3\cos 2\nu t - 1)$$
$$\beta_{13} = 0, \quad \beta_{14} = -{}^1/_2\nu^{-1}(1 - e\cos\nu t), \quad \beta_{23} = \nu^{-1}(1 - e\cos\nu t), \quad \beta_{24} = 0$$
$$\beta_{33} = {}^1/_2\nu^{-1}[\cos 2\nu t - e\,(3\sin\nu t\sin 2\nu t + \cos\nu t)] \qquad (5.16)$$
$$\beta_{34} = {}^1/_2\nu^{-1}[\sin 2\nu t + e\sin\nu t\,(3\cos 2\nu t - 1)]$$
$$\beta_{43} = -{}^1/_2\nu^{-1}[\sin 2\nu t + e\sin\nu t\,(3\cos 2\nu t - 1)]$$
$$\beta_{44} = {}^1/_2\nu^{-1}[\cos 2\nu t - e\,(3\sin\nu t\sin 2\nu t + \cos\nu t)]$$
$$C_1{}^\circ = (1+e)\,\delta x^\circ - {}^1/_2\nu^{-1}(1-e)\,\delta z^{\circ\cdot}, \quad C_2{}^\circ = (1+e)\,\delta z^\circ + {}^1/_2\nu^{-1}(1-e)\,\delta x^{\circ\cdot}$$
$$C_3{}^\circ = {}^1/_2\nu^{-1}(1-e)\,\delta x^{\circ\cdot}, \quad C_4{}^\circ = {}^1/_2\nu^{-1}(1-e)\,\delta z^{\circ\cdot}$$

If it is assumed that the instrument errors are constant, substitution of (5.16) into (5.14) and integration leads to the following expressions for δx and δz:

$$\delta x = {}^1/_4\nu^{-2}\Delta n_x(1 - \cos 2\nu t) + ({}^1/_4\nu^{-2}\Delta n_z + {}^3/_2\Delta r)(\sin 2\nu t - 2\nu t) +$$
$$+ \delta x^\circ + {}^1/_2\nu^{-1}\delta x^{\circ\cdot}\sin 2\nu t + {}^1/_2\nu^{-1}\delta z^{\circ\cdot}(\cos 2\nu t - 1) + e\,[{}^1/_2\nu^{-2}\Delta n_x(-{}^5/_4\cos\nu t +$$
$$+ 2\cos 2\nu t - {}^3/_4\cos 3\nu t) + {}^1/_2\nu^{-2}\Delta n_z(\nu t\cos\nu t + {}^3/_4\sin\nu t - 2\sin 2\nu t + {}^3/_4\sin 3\nu t) +$$
$$+ {}^5/_3\nu^{-1}a\,\Delta m_y(-2\sin\nu t + \sin 2\nu t) + \Delta r\,({}^3/_2\nu t\cos\nu t - {}^{39}/_8\sin\nu t + {}^9/_8\sin 3\nu t) +$$
$$+ \delta x^\circ(1 - \cos\nu t) + {}^1/_2\nu^{-1}\delta z^{\circ\cdot}(1 - \cos 2\nu t - 3\sin\nu t\sin 2\nu t) +$$
$$+ {}^1/_2\nu^{-1}\delta x^{\circ\cdot}(-\sin\nu t - \sin 2\nu t + 3\sin\nu t\cos 2\nu t)]$$

$$\delta z = {}^1/_4\nu^{-2}\Delta n_x(2\nu t - \sin 2\nu t) + ({}^1/_4\nu^{-2}\Delta n_z + {}^3/_4\Delta r)(1 - \cos 2\nu t) + \delta z^\circ +$$
$$+ {}^1/_2\nu^{-1}\delta x^{\circ\cdot}(1 - \cos 2\nu t) + {}^1/_2\nu^{-1}\delta z^{\circ\cdot}\sin 2\nu t + e\,[{}^1/_2\nu^{-2}\Delta n_x(-\nu t\cos\nu t - {}^3/_4\sin\nu t +$$
$$+ 2\sin 2\nu t - {}^3/_4\sin 3\nu t) + {}^1/_2\nu^{-2}\Delta n_z(-{}^5/_4\cos\nu t + 2\cos 2\nu t - {}^3/_4\cos 3\nu t) +$$
$$+ \nu^{-1}a\,\Delta m_y(-1 + {}^5/_3\cos\nu t - {}^5/_6\cos 2\nu t + {}^1/_6\cos 3\nu t) + {}^9/_8\Delta r\,(\cos\nu t - \cos 3\nu t) +$$
$$+ \delta z^\circ(1 - \cos\nu t) + {}^1/_2\nu^{-1}\delta x^{\circ\cdot}(-1 + \cos 2\nu t + 3\sin\nu t\sin 2\nu t) +$$
$$+ {}^1/_2\nu^{-1}\delta z^{\circ\cdot}(-\sin 2\nu t - \sin\nu t + 3\sin\nu t\cos 2\nu t)]$$

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